Ricardian Trade Models:
Part II: Eaton and Kortum (2002)

ECON 871

Eaton and Kortum (EK) (2002): Motivation

- Eaton and Kortum (2002) (EK) brought about a “Ricardian Revival,” building a Ricardian model rich enough for quantitative analysis.
- Ricardian model of trade (i.e., trade is generated by differences in technology), extended to a multi-good and multi-country setup.
- Use a probabilistic formulation of productivity differences and show how the model links bilateral trade flows to geography and prices.
- Small number of parameters, so well-suited for quantitative work.

EK 2002: Setup

- There is a continuum of goods $j \in [0, 1]$.
- Let $Z_i(j)$ denote the amount of good j that a bundle of inputs can produce in country i.
  - The cost of producing a unit of good j in country i is then:

$$\frac{c_i}{Z_i(j)}$$

    where c_i is the cost of a bundle of inputs.

- Consumers have CES preferences over goods with elasticity of substitution σ:

$$U_i = \left[\int_0^1 Q(j)^{\frac{(\sigma-1)}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}$$

EK 2002: Setup

- There are **iceberg trade costs**: $d_{ni} > 1$ units have to be shipped from country i to country n for one unit to arrive.
  - Normalize $d_{ii} = 1$.
  - Assume the "triangle inequality" holds: $d_{ni} \leq d_{nk} d_{ki}$.

- The cost of obtaining good j from country i in country n is then given by:

$$p_{ni}(j) = \frac{c_i d_{ni}}{Z_i(j)}$$

- Markets are perfectly competitive, implying that:

$$p_n(j) = \min_i \{p_{ni}(j)\}$$

  This says, country n will buy good j from the cheapest possible source, i.

EK 2002: Technology

The key tractability assumption is that the $Z_i(j)$'s are realizations of random variables.

- Need a distribution function for which we can compute certian order statistics—in particular, the distribution of the min (or max).

- So, assume that for each good j, the distribution for country i's productivity is **Frechet**:

$$\Pr\left[Z_i(j) \leq z\right] = F_i(z) = e^{-T_i z^{-\theta}}$$

  - $T_i > 0$ governs the location of the productivity distribution for country i. Higher $T_i \implies$ higher productivity draw is more likely for any good j. "Absolute advantage."

  - $\theta > 1$ determines the *dispersion* of productivity, where a higher θ means there is less dispersion (common across countries). "Comparative advantage."

Why Frechet?

The **Frechet distribution** is an Extreme Value (type II) distribution that is *max stable*.

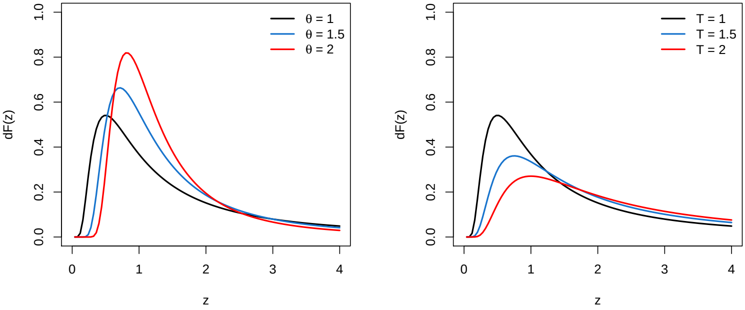
- Suppose $Z_1, Z_2, ..., Z_N$ follow Frechet(T_i, θ) distributions.
- Define $Z_{\max} = max\{Z_1, Z_2, ..., Z_N\}$.
- Then, $F_{\max}(z) = e^{-\sum_{i=1}^{N} T_i z^{\theta}} = e^{-z^{\theta}\sum_{i=1}^{N} T_i}$.
- Therefore, $Z_{\max} \sim$ Frechet($\sum_{i=1}^{N} T_i, \theta$)

This property is useful for **environments with perfect competition** because we can characterize the productivity of the most productive (lowest price) producers.

Similar idea to using Pareto distributions for studying extensive margins, which are still Pareto if you truncate the left tail.

Why Frechet?

Figure: Frechet Distribution, Varying θ and T



The Price Distribution

Let's start by writing down the distribution of prices in country n.

- Let $P_{ni}(Z_i) \equiv c_i d_{ni} / Z_i$ be the unit cost at which country i can sell good Z to country n.
  I've dropped the j's since goods are symmetric except for productivity.
- Then, we can define the distribution of prices presented to country n by country i as:

$$
\begin{aligned}
G_{ni}(p) \equiv \Pr(P_{ni}(Z_i) < p) &= \Pr(Z_i \geq c_i d_{ni}/p) \\
&= 1 - F_i(c_i d_{ni}/p) \\
&= 1 - e^{-T_i (c_i d_{ni})^{-\theta} p^{\theta}}
\end{aligned}
$$

Distribution of Prices

From the last slide, we have:

$$G_{ni}(p) = 1 - e^{-T_i (c_i d_{ni})^{-\theta} p^{\theta}}$$

The lowest price for a good in country *n* will be less than *p* unless each source's price is greater than *p*. Hence, the distribution of prices for what *n actually buys* is:

$$\begin{aligned} G_n(p) &\equiv \Pr\left[P_n(Z) \leq p\right] \\ &= 1 - \prod_{i=1}^{N} \left[1 - G_{ni}(p)\right] \\ &= 1 - e^{-\Phi_n p^{\theta}} \end{aligned}$$

where $\Phi_n = \sum_{i=1}^{N} T_i (c_i d_{ni})^{-\theta}$.

Distribution of Prices

So the distribution of prices in country n is given by:

$$G_n(p) = 1 - e^{-\Phi_n p^\theta}$$

Unpacking this a little bit:

- Φ_n is a country specific price parameter.
- T_i indexes how productive country i is (on average).
  Prices tend to be lower as T_i's get higher.
- c_i is how costly inputs are in country i.
  Prices tend to be lower as c_i's and $d'_{ni}s$ are lower.
- d_{ni} are the iceberg costs when shipping from i to n.
  More "remote" countries will have higher prices.
- In autarky, $d_{ni} \to \infty \; \forall \; i \neq n \implies \Phi_n = T_n c_n^{-\theta}$

EK 2002: The Allocation of Purchases

Consider a particular good. **Country n buys the good from country i if country i is the lowest-cost producer.**

The probability of this event turns out to be country i's contribution to country n's price parameter, Φ_n:

$$\pi_{ni} = \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n}$$

This comes from the following:

$$\pi_{ni} = \int_0^\infty \underbrace{\prod_{s=1, s\neq i}^{N} \Pr(P_{ns} > p)}_{\substack{\text{Prob. no other} \\ \text{country offers price} < p}} \overbrace{d\Pr(P_{ni} \leq p)}^{\substack{\text{Prob. country } i \text{ offers} \\ \text{a price} \leq p}}$$

Math

The Conditional Price Distribution

Earlier, we wrote that $G_n(p)$ is the distribution of prices in any country n:

$$G_n(p) = 1 - e^{-\Phi_n p^\theta}$$

It turns out, the price of a good that country n actually buys from any country i also has the distribution $G_n(p)$.

To see this, note that if country n buys a good from country i, it means that i is the least cost producer.

- Let the price at which country i sells this good in country n be given by q.
- Then, the probability that i is the least-cost supplier is:

$$\prod_{s \neq i} \Pr(P_{ni} \geq q) = \prod_{s \neq i} [1 - G_{ns}(q)] = e^{-\Phi_n^{-i} q^\theta}$$

Conditional Price Distribution

From the last slide, we have the probability that i, selling at price q, is the least cost supplier is:

$$\prod_{s \neq i} \Pr(P_{ni} \geq q) = e^{-\Phi_n^{-i} q^{\theta}}$$

- Then, the **joint probability that country** i **has a unit cost** q of delivering the good to country n *and* that i **is the least cost supplier** is:

$$e^{-\Phi_n^{-i} q^{\theta}} \times dG_{ni}(q)$$

Conditional Price Distribution

Integrating this probability ($e^{-\Phi_n^{-i} q^\theta} \times dG_{ni}(q)$) over all prices $q \leq p$ and plugging in $G_{ni}(q) = 1 - e^{-T_i (c_i d_{ni})^{-\theta} p^\theta}$, we have:

$$
\begin{aligned}
& \int_0^p e^{-\Phi_n^{-i} q^\theta} dG_{ni}(q) \\
= & \int_0^p e^{-\Phi_n^{-i} q^\theta} \theta T_i (c_i d_{ni})^{-\theta} q^{\theta - 1} e^{-T_i (c_i d_{ni})^{-\theta} p^\theta} dq \\
= & \left(\frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n} \right) \int_0^p e^{-\Phi_n q^\theta} \theta \Phi_n q^{\theta - 1} dq \\
= & \pi_{ni} G_n(p)
\end{aligned}
$$

Conditional Price Distribution

Next, note that π_{ni} is defined as the probability that for any particular good, country i is the least cost supplier in n.

Thus, the *conditional distribution* of the price charged by i in n for the goods that i actually sells in n is:

$$\frac{1}{\pi_{ni}} \int_0^p e^{-\Phi_n^{-i} q^\theta} dG_{ni}(q) = G_n(p)$$

The fact that the conditional distribution is the same as the unconditional distribution is a property of the Frechet distribution.

Conditional Price Distribution

The key implications to take away from all of this are as follows:

1. Countries that have higher c's, d's, or lower T's, sell a **smaller range** of goods, but charge the **same average prices**.
   Adjustment is at the extensive margin.

2. On the flip side, a source with a higher state of technology, lower input cost, or lower barriers exploits its advantage by selling a wider range of goods, exactly to the point at which the distribution of prices for what it sells in n is the same as n's overall price distribution.

3. The **share of spending by country n on goods from country i is also** π_{ni}. That is:

$$\frac{X_{ni}}{X_n} = \pi_{ni} = \frac{T_i(c_i d_{ni})^{-\theta}}{\sum_i T_i(c_i d_{ni})^{-\theta}}$$

CES Price Index

The CES price index can be derived as:

$$p_n = \left(\int_0^1 (p_n(j))^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} = \gamma(\Phi_n)^{-\frac{1}{\theta}}$$

Where:

$$\gamma = \left[\Gamma \left(\frac{\theta + 1 - \sigma}{\theta} \right) \right]^{\frac{1}{1-\sigma}}$$

and $\Gamma[t]$ is the Gamma function:

$$\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$$

▸ Derivation

Taking Stock So Far

So far, we've exploited properties of the price (Frechet) distribution to establish the following:

1. The probability that country i supplies good j to country n is:

   $$\pi_{ni} = T_i(c_i d_{ni})^{-\theta}/\Phi_n$$

   This is also the fraction of goods that n buys from i.
2. The distribution of prices in country n, $G_n(p)$ applies to goods actually purchased, **without regard to their source**.
3. The exact price index for U is $p_n = \gamma \Phi_n^{-\frac{1}{\theta}}$, for $\sigma < 1 + \theta$.
4. From (2), expenditure per good does not differ by source, and thus we have:

   $$\frac{X_{ni}}{X_n} = \pi_{ni} = \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n}$$

   where $X's$ are expenditures.

Aggregate Trade Flows

At this point, we can also solve for aggregate trade flows.

Country *i*'s total production is:

$$Q_i = \sum_{m=1}^{N} X_{mi} = T_i (c_i)^{-\theta} \sum_{m=1}^{N} \frac{d_{mi}^{-\theta} X_m}{\Phi_m}$$

Plugging this into our expression from the last slide for X_{ni}/X_n and plugging in for $\Phi_n = (p_n)^{-\theta}\gamma^{\theta}$, we get:

$$X_{ni} = \frac{\left(\frac{d_{ni}}{p_n}\right)^{-\theta} X_n}{\sum_{m=1}^{N} \left(\frac{d_{mi}}{p_m}\right)^{-\theta} X_m} Q_i$$

Gravity in Trade Flows

If we take logs of the equation on the previous side, we get something that looks a lot like a gravity regression:

$$\ln X_{ni} = \underbrace{-\ln\left(\sum_{m=1}^{N}\left(\frac{d_{mi}}{p_m}\right)^{-\theta} X_m\right)}_{\text{Country } i \text{ f.e.}} + \overbrace{\theta \ln p_n}^{\text{Country } n \text{ f.e.}} - \theta \underbrace{\ln d_{ni}}_{\text{distance}} + \overbrace{\ln X_n + \ln Q_i}^{\text{Country } i \text{ and } n\text{'s sizes.}}$$

- Gross output, X_n and Q_i, enters linearly (unit elasticities).
- Distance is regulated by $-\theta$—more dispersion in productivity (lower θ), more likely goods will travel farther distances.

Closing the Model

We still need to close the model and solve for factor prices.

- To make the model more realistic, EK stipulate the production is Cobb Douglas in labor and a basket of intermediate goods.
- Intermediate goods are a CES bundle of the same goods that are consumed.
  Implicitly, this means input costs will now be affected by trade.
- This implies that the cost of an input bundle in country i will be equal to:

$$c_i = w_i^{\beta} p_i^{1-\beta}$$

  where w_i is wages and p_i is the same CES price index we derived earlier.

Real Wages

Next, we can write down an expression for real wages. Start with the price index, p_i:

$$p_n = \gamma\left(\Phi_n\right)^{-\frac{1}{\theta}} \implies \Phi_i = (p_i)^{-\theta}\gamma^{\theta}$$

From our expression for π_{ni}, we can write down the expression for the domestic share of consumption, π_{ii} as:

$$\frac{X_{ni}}{X_n} = \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n} \implies \pi_{ii} = \frac{T_i(c_i)^{-\theta}}{\Phi_i}$$

Plugging in for c_i and Φ_i, we have:

$$\pi_{ii} = \frac{T_i(w_i^{\beta} p_i^{1-\beta})^{-\theta}}{(p_i)^{-\theta}\gamma^{\theta}} \implies \frac{w_i}{p_i} = \gamma^{-\frac{1}{\beta}}\left(\frac{T_i}{\pi_{ii}}\right)^{\frac{1}{\beta\theta}}$$

Equilibrium Prices

Next, we can plug in input costs to the price level to get an expression for the price level:

$$p_n = \gamma(\Phi_n)^{-\frac{1}{\theta}} = \gamma \left(\sum_{i=1}^{N} T_i (c_i d_{ni})^{-\theta} \right)^{-\frac{1}{\theta}}$$

$$= \gamma \left(\sum_{i=1}^{N} T_i (w_i^{\beta} p_i^{1-\beta} d_{ni})^{-\theta} \right)^{-\frac{1}{\theta}}$$

Given w_i, this generally neesd to be solved numerically. We'll go through a couple of special cases. We can also plug in input costs here:

$$\frac{X_{ni}}{X_n} = \pi_{ni} = T_i \left(\frac{\gamma d_{ni} w_i^{\beta} p_i^{1-\beta}}{p_n} \right)^{-\theta}$$

Labor Market Equilibrium

Labor income is equal to labor's share of the value of output:

$$w_i L_i = \beta Q_i = \beta \sum_{i=1}^{N} X_{ni} = \beta \sum_{n=1}^{N} \pi_{ni} X_n$$

Total expenditures in country n are then:

$$X_n = \underbrace{\frac{1-\beta}{\beta} w_n L_n}_{\text{Intermediates}} + \underbrace{w_n L_n}_{\text{Final consumption}} = \frac{1}{\beta} w_n L_n$$

Wages, therefore, satisfy:

$$w_i L_i = \sum_{n=1}^{N} \pi_{ni} w_n L_n$$

Solving for the Equilibrium

The equilibrium is pinned down by three sets of equations:

Wages:

$$w_i L_i = \sum_{n=1}^{N} \pi_{ni} w_n L_n, \quad i = 1, 2, ..., N$$

Trade Shares:

$$\pi_{ni} = T_i \left(\frac{\gamma d_{ni} w_i^{\beta} p_i^{1-\beta}}{p_n} \right)^{-\theta}, \quad i, n = 1, 2..., N$$

Prices:

$$p_n = \gamma \left[\sum_{i=1}^{N} T_i (d_{ni} w_i^{\beta} p_i^{1-\beta})^{-\theta} \right]^{-\frac{1}{\theta}}, \quad n = 1, 2, ..., N$$

Special Case: Free Trade

EK show that in a **zero gravity** or **free trade** world, where all $d_{ni} = 1$, we can solve for real GDP (welfare) in closed form:

$$\frac{w_i}{p_i} = \gamma^{-\frac{1}{\beta}} T_i^{\frac{1}{(1+\theta\beta)}} L_i^{\frac{\theta\beta}{1+\theta\beta}} \left[\sum_{i=1}^{N} T_k^{\frac{1}{1+\theta\beta}} L_k^{\frac{\theta\beta}{1+\theta\beta}} \right]^{\frac{1}{\theta\beta}}$$

This is a little messy, but we can see the following:

- Holding L_i constant, welfare is increasing in T_k *everywhere*.
- An increase in T at home confers an extra benefit because it raises home wages relative to wages abroad.
- How much i benefits from an increase in T_k depends on the size of k's labor force relative to i's. Small countries benefit more from technological improvements abroad.

Special Case: Autarky

EK also solve for welfare in autarky by goign back to the expression for real wages in country i:

$$\frac{w_i}{p_i} = \gamma^{-\frac{1}{\beta}} \left(\frac{T_i}{\pi_{ii}} \right)^{\frac{1}{\beta\theta}}$$

and setting $\pi_{ii} = 1$. This yields:

$$\frac{w_i}{p_i} = \gamma^{-\frac{1}{\beta}} T_i^{\frac{1}{\theta\beta}}$$

So, in autarky, welfare is simply increasing in own-country technology, T_i.

Gains from Trade

We can rewrite the free trade welfare formula to see that countries are better off under trade than autarky:

$$
\begin{aligned}
\frac{w_n}{p_n} &= \gamma^{-\frac{1}{\beta}} T_n^{\frac{1}{1+\beta}} \left[\sum_{i=1}^{N} T_i^{\frac{1}{1+\theta\beta}} (L_i/L_N)^{\frac{\theta\beta}{1+\theta\beta}} \right]^{\frac{1}{\theta\beta}} \\
&= \underbrace{\gamma^{-\frac{1}{\beta}} T_n^{\frac{1}{\theta\beta}}}_{\text{Autarky}} \underbrace{\left[\sum_{i=1}^{N} (T_i/T_n)^{\frac{1}{1+\theta\beta}} (L_i/L_n)^{\frac{\theta\beta}{1+\theta\beta}} \right]^{\frac{1}{\theta\beta}}}_{>1}
\end{aligned}
$$

Uncovering Model Parameters

In order to perform counterfactuals, we still need to estimate several parameters.

EK do this in two steps:

1. **First, estimate θ.** Several ways to do this depending on data availability. We'll go over the following:
   - EK (baseline)
   - Simanovska and Waugh
   - Caliendo and Parro
2. **Given θ, estimate T_i and d_{ni}.**

Estimating θ: EK Baseline

Going back to our original expression for π_{ni}:

$$\frac{X_{ni}}{X_n} = \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n}$$

Divide through by X_{ii}/X_i to get:

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \frac{\Phi_i}{\Phi_n} d_{ni}^{-\theta} = \left(\frac{p_i d_{ni}}{p_n}\right)^{-\theta}$$

With the right data, this structural relationship between (normalized) import shares and prices can be estimated to get θ.

Estimating θ: EK Baseline

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left(\frac{p_i d_{ni}}{p_n}\right)^{-\theta}$$

EK construct the LHS using data on bilateral trade in manufactures among 19 OECD countries in 1990.

- 342 observations (X_{ni}'s).

To estimate $\ln(p_i d_{ni}/p_n)$, they use data on retail prices in the 19 countries on 50 manufactured products.

- For each country-pair (n, i) and good j, calculate the log *relative* price: $r_{ni}(j) = \ln p_n(j) - \ln p_i(j)$.
- The log of p_i/p_n is then the mean across the j's of $r_{ni}(j)$.
- For d_{ni}, use the model's prediction that, for any commodity j, $r_{ni}(j)$ is bounded above by $\ln(d_{ni}$, with this bound attained for goods that n imports from i.

Estimating θ: EK Baseline

Every country in EK's sample imports from every other country, so they take the (second) highest value of r_{ni} across commodities to obtain a measure of $\ln(d_{ni})$.

- Use the second highest in case of possible measurement error in the prices for particular commodities.

So, ultimately, they measure $\ln(p_i d_{ni}/p_n)$ by the term D_{ni}, defined as:

$$D_{ni} = \frac{\max 2_j \{r_{ni}(j)\}}{\sum_{j=1}^{50} [r_{ni}(j)] / 50}$$

The price measure $e^{D_{ni}}$ reflects what the price index in n would be for a buyer there who insisted on buying everything from i, relative to the actual price index in n (the price index for a buyer purchasing each good from the cheapest source).

Estimating θ: EK Baseline

With D_{ni} in hand, we can now estimate θ from:

$$S_{ni} = \ln\left(\frac{X_{ni}/X_n}{X_{ii}/X_i}\right) = -\theta D_{ni}$$

- Use method of moments to get an estimate of $\hat{\theta} = 8.28$.
- This θ — the elasticity of trade flows to trade barriers is often referred to as **"the"** trade elasticity.
- We'll cover some other ways of estimating this parameter throughout the semester.

Uncovering Other Parameters

EK show some other ways to estimate θ as well, but we'll stick with the baseline. With this in hand, we can write down another version of the gravity equation:

$$\frac{X_{ni}}{X_{nn}} = \frac{\pi_{ni}}{\pi_{nn}} = \frac{T_i}{T_n}\left(\frac{w_i}{w_n}\right)^{-\theta\beta}$$

To avoid having to measure relative price levels:

$$\frac{p_i}{p_n} = \frac{w_i}{w_n}\left(\frac{T_i}{T_n}\right)^{-1/\theta\beta}\left(\frac{X_i}{X_{ii}}X_n/X_{nn}\right)^{-1/\theta\beta}$$

Combining and rearranging:

$$\ln\frac{X'_{ni}}{X'_{nn}} = -\theta\ln d_{ni} + \frac{1}{\beta}\ln\frac{T_i}{T_n} - \theta\ln\frac{w_i}{w_n}$$

where $\ln X'_{ni} \equiv \ln X_{ni} - [(1-\beta)/\beta]\ln(X_i/X_{ii})$

Uncovering Other Parameters

Letting $S_i \equiv \frac{1}{\beta} \ln T_i - \theta \ln w_i$, this becomes:

$$\ln \frac{X'_{ni}}{X'_{nn}} = -\theta \ln d_{ni} + S_i - S_n$$

- Estimate this equation using gravity variables for d_{ni} (proximity, language, treaties).
- S_i and S_n are country fixed effects.
- $\beta = 0.21$ (average labor share in gross manufacturing).
- Using wage data and $\theta = 8.28$, recover the T_i from the S_i.
- Using $\theta = 8.28$, can recover the d_{ni} from coefficients on gravity variables.

See Section 5.1 for more detail.

Counterfactuals

With all the parameters in hand, EK then goes through a bunch of counterfactuals.

- **Experiment 1:** Let $d_{ni} \to \infty$. (i.e., Autarky)
- **Experiment 2:** Let $d_{ni} \to 1$. (i.e., Free Trade)
- **Experiment 3:** Smooth decline of trade barriers with mobile labor.
- **Experiment 4:** Increase in foreign technology.
- **Experiment 5:** Eliminating tariffs. Extend the model to account for tariff revenues.

Counterfactuals

Experiment 1: Moving to autarky.

- Smaller coutnries tend to be hurt more.

TABLE IX

THE GAINS FROM TRADE: RAISING GEOGRAPHIC BARRIERS

	Percentage Change from Baseline to Autarky					
	Mobile Labor			Immobile Labor		
Country	Welfare	Mfg. Prices	Mfg. Labor	Welfare	Mfg. Prices	Mfg. Wages
Australia	−1.5	11.1	48.7	−3.0	65.6	54.5
Austria	−3.2	24.1	3.9	−3.3	28.6	4.5
Belgium	−10.3	76.0	2.8	−10.3	79.2	3.2
Canada	−6.5	48.4	6.6	−6.6	55.9	7.6
Denmark	−5.5	40.5	16.3	−5.6	59.1	18.6
Finland	−2.4	18.1	8.5	−2.5	27.9	9.7
France	−2.5	18.2	8.6	−2.5	28.0	9.8
Germany	−1.7	12.8	−38.7	−3.1	−33.6	−46.3
Greece	−3.2	24.1	84.9	−7.3	117.5	93.4
Italy	−1.7	12.7	7.3	−1.7	21.1	8.4
Japan	−0.2	1.6	−8.6	−0.3	−8.4	−10.0
Netherlands	−8.7	64.2	18.4	−8.9	85.2	21.0
New Zealand	−2.9	21.2	36.8	−3.8	62.7	41.4
Norway	−4.3	32.1	41.1	−5.4	78.3	46.2
Portugal	−3.4	25.3	25.1	−3.9	53.8	28.4
Spain	−1.4	10.4	19.8	−1.7	32.9	22.5
Sweden	−3.2	23.6	−3.7	−3.2	19.3	−4.3
United Kingdom	−2.6	19.2	−6.0	−2.6	12.3	−6.9
United States	−0.8	6.3	8.1	−0.9	15.5	9.3

Notes: All percentage changes are calculated as $100\ln(x'/x)$ where x' is the outcome under autarky ($d_{ni} \to \infty$ for $n \neq i$) and x is the outcome in the baseline.

Counterfactuals

Experiment 1: Moving to zero-gravity.

- Still a long way from a zero-gravity world.

TABLE X

THE GAINS FROM TRADE: LOWERING GEOGRAPHIC BARRIERS

	Percentage Changes in the Case of Mobile Labor					
	Baseline to Zero Gravity			Baseline to Doubled Trade		
Country	Welfare	Mfg. Prices	Mfg. Labor	Welfare	Mfg. Prices	Mfg. Labor
Australia	21.1	−156.7	153.2	2.3	−17.1	−16.8
Austria	21.6	−160.3	141.5	2.8	−20.9	41.1
Belgium	18.5	−137.2	69.6	2.5	−18.6	68.8
Canada	18.7	−139.0	11.4	1.9	−14.3	3.9
Denmark	20.7	−153.9	156.9	2.9	−21.5	72.6
Finland	21.7	−160.7	172.1	2.8	−20.9	44.3
France	18.7	−138.3	−7.0	2.3	−16.8	15.5
Germany	17.3	−128.7	−50.4	1.9	−14.3	12.9
Greece	24.1	−178.6	256.5	3.3	−24.8	29.6
Italy	18.9	−140.3	6.8	2.2	−16.1	5.7
Japan	16.6	−123.5	−59.8	0.9	−6.7	−24.4
Netherlands	18.5	−137.6	67.3	2.5	−18.5	65.6
New Zealand	22.2	−164.4	301.4	2.8	−20.5	50.2
Norway	21.7	−161.0	195.2	3.1	−22.9	69.3
Portugal	22.3	−165.3	237.4	3.1	−22.8	67.3
Spain	20.9	−155.0	77.5	2.4	−18.0	−4.4
Sweden	20.0	−148.3	118.8	2.7	−19.7	55.4
United Kingdom	18.2	−134.8	3.3	2.2	−16.4	28.5
United States	16.1	−119.1	−105.1	1.2	−9.0	−26.2

Notes: All percentage changes are calculated as $100\ln(x'/x)$ where x' is the outcome under lower geographic barriers and x is the outcome in the baseline.

A Few Extensions: Multiple Sectors

- Costinot, Donaldson, & Komunjer (2012) extend EK to a multi-country **multi-sector** world.
- K industries in each country $(k = 1, ..., K)$, and within each industry, a continuum of varieties is produced.
- With multiple sectors, T_i now has a sector dimension, but θ remains common across all countries.
- Model predicts what are the goods that a country will specialize in.
- For any importer j and any pair of exporters, $i, i' \neq j$, the ranking of relative fundamental productivities determines the ranking of exports:

$$\frac{T_i^1}{T_{i'}^1} \leq ... \leq \frac{T_i^K}{T_{i'}^K} \iff \frac{X_{ji}^1}{X_{ji'}^1} \leq ... \leq \frac{X_{ji}^K}{X_{ji'}^K}$$

A Few Extensions: Imperfect Competition

- Bernard, Eaton, Jensen, and Kortum (BEJK, 2003) extend EK to allow for **imperfect competition** between varieties.
- Consumer prices are above marginal costs.
- Model predicts a distribution of markups in each market, that is bounded above by the Dixit-Stiglitz (CES) constant markup.
- Additional predicitons on within-country heterogeneity in prices, productivities, etc.

Appendix Slides

Allocation of Purchases

$$
\begin{aligned}
\pi_{ni} &= \int_0^{\infty} \prod_{s=1, s\neq i}^{N} [1 - G_{ns}(p)]\, dG_{ni}(p) \\
&= \int_0^{\infty} \prod_{s=1, s\neq i}^{N} \left[e^{-T_s (c_s d_{ns})^{-\theta} p^{\theta}} \right] \left[(T_i (c_i d_{ni})^{-\theta} \theta p^{\theta-1}) e^{-T_i (c_i d_{ni})^{-\theta} p^{\theta}} \right] dp \\
&= (T_i (c_i d_{ni})^{-\theta}) \int_0^{\infty} \prod_{s=1}^{N} \left[e^{-T_s (c_s d_{ns})^{-\theta} p^{\theta}} \right] \theta p^{\theta-1} dp \\
&= (T_i (c_i d_{ni})^{-\theta}) \int_0^{\infty} e^{-\Phi_n p^{\theta}} \theta p^{\theta-1} dp \\
&= (T_i (c_i d_{ni})^{-\theta}) \left[-\frac{1}{\Phi_n} e^{-\Phi_n p^{\theta}} \right]_{p=0}^{\infty} = (T_i (c_i d_{ni})^{-\theta}) \left[0 - \frac{1}{\Phi_n} e^{0} \right] \\
&= \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n}
\end{aligned}
$$

Return

Derivation of the Exact Price Index

To show this, note that:

$$
\begin{aligned}
p_n^{1-\sigma} &= \int_0^1 p_n(u)^{1-\sigma} du \\
&= \int_0^\infty p^{1-\sigma} dG_n(p) \\
&= \int_0^\infty p^{1-\sigma} \Phi_n \theta p^{\theta-1} e^{-\Phi_n p^\theta} dp
\end{aligned}
$$

Then, let $x = \Phi_n p^\theta$. This gives us:

$$
dx = \Phi_n \theta p^{\theta-1} \qquad \text{and} \qquad p^{1-\sigma} = (x/\Phi_n)^{(1-\sigma)/\theta}
$$

Derivation of the Exact Price Index

Plugging this back in, we have:

$$\begin{aligned} p_n^{1-\sigma} &= \int_0^\infty (x/\Phi_n)^{(1-\sigma)/\theta} e^{-x} dx \\ &= \Phi_n^{-(1-\sigma)/\theta} \int_0^\infty x^{(1-\sigma)/\theta} e^{-x} dx \\ &= \Phi_n^{-(1-\sigma)/\theta} \Gamma\left(\frac{1-\sigma}{\theta} + 1\right) \end{aligned}$$

This implies $p_n = \gamma \Phi_n^{-1/\theta}$, with $\frac{1-\sigma}{\theta} + 1 > 0$ or $\sigma - 1 < \theta$ for the gamma function to be well-defined.

Return